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DISTRIBUTION OF THE PARAMETERS OVER THE VERTICAL IN A BREAK-THROUGH WAVE

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We investigate the distribution of the parameters of flow over the vertical in a break-through wave.

Different approaches in the theory of a break-through wave are considered in [1], but here we investigate a problem that up to now has not been studied in the literature.

1. General Statements. In recent years, in connection with increased construction of high-pressure hydroelectric structures, investigations of open water flows with relatively high velocities (20–30 m/sec and higher) are being intensively developed. One of the specific features of such processes is the occurrence of aeration, which is a spontaneous trapping of air by a water flow.

By now, many facts have been accumulated that are indicative of the great influence of air bubbles on the structure of turbulence in a stream channel. The investigations of F. I. Frankl, G. I. Barenblatt, and others, which contain a thorough and sound approach to the mechanism of turbulent motion of a two-component medium, are at the same time very complex and do not allow one to obtain practical results. The contemporary hydrodynamics of multiphase systems [2-5] is also far from aerated flows. There is a theory of gas-liquid flows, but it is for motion in a tube. Here, one of the unsolved probelms of the theory of aerated flows is the vertical distribution of the parameters of already formed (developed) aerated flow. Below, we undertook investigation of this problem.

We assume that a break-through wave is an aerated open flow, i.e., a mixture of water with air (Fig. 1). Usually, the fact of suction of air by the water stream in the same break-through wave is ignored. Below, we suggest an approach that takes account of the fact of the presence of air in an open flow.

The main problem of investigations is determination of the shape and size of the break-through wave and the distributions of velocity and density over its thickness, width, and length. If a moving break-through wave is considered as a flow of a certain continuous medium (almost always turbulent), then this information can be obtained in principle. But no equations are as yet available that would give the distribution of its parameters over all three spatial coordinates at each instant of time and would constitute a complete system (i.e., a system in which the number of equations is sufficient for their solution from the prescribed external conditions). They are absent even for turbulent flows of water, whereas a break-through wave is a much more complex phenomenon.

To calculate a break-through wave, it is possible to use complete systems of two types:

1. Equations are constructed on the assumption that the change in the parameters over the length and width of the break-through wave can be neglected. Using these equations, if is possible to find the distribution of velocity and density over the thickness of the moving layer. However, then the presence of the leading front is ignored, and its motion is not calculated, so that these equations describe the motion not of a break-through wave, but rather of a certain infinite flow of a water-air mixture.

2. Hydraulic equations for the break-through wave average all the parameters used over the thickness of the moving layer, i.e., the distribution of the thickness-averaged velocities is described along and across the break-through wave. Models of this type are considered in [1]. At the present stage of investigations it seems that the most promising models for calculating the characteristics of a break-through wave are hydraulic ones.

2. Formulation of the Problem of the Distribution of the Parameters of a Flow over the Thickness. We will consider one of the models that refer to the first type, i.e., we will use complete (not averaged over the thickness)

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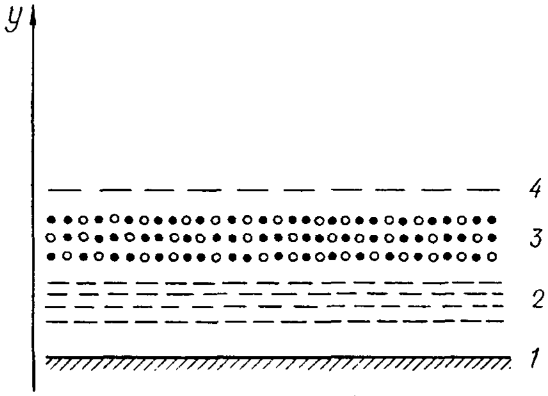

Fig. 1. Scheme of motion of an aerated flow: 1) ground or structure, 2) water, 3) aerated flow, 4) air.

equations of the hydraulics of turbulent flows of mixtures of different components (water and air). To study the motion of an aerated liquid, a simplified model is suggested in which water and air are considered not separately, but as a continuous medium of variable density.

In accordance with the conservation laws, the density ρ_{mix} and velocity v_{mix} of this type of medium (a mixture) can be represented in the following way:

$$\rho_{mix} = \rho_{liq}(1 - s) + \rho_0 s\,;\ \ \rho_{mix}v_{mix} = \rho_{liq}(1 - s)\,v_{liq} + \rho_0 s v\,.$$

For an aerated flow $s \ll 1$. Therefore we have

$$\rho_{mix} = \rho_{liq} + \rho_0 s\,;\ \ \rho_{mix}v_{mix} = \rho_{liq}v_{liq} + \rho_0 s v\,. \tag{1}$$

Below, for convenience instead of the function s we will consider $\rho = \rho_0 s$, i.e., the mass of air per unit volume of the mixture. Then $\rho_{mix} = \rho_{liq} + \rho$. The liquid is considered to be incompressible, i.e., $\rho_{liq} = \text{const}$. The velocities of the liquid and the air particles in the mixture are different. To determine them, it is necessary to study the motion of each component separately, and this greatly complicates the problem. Therefore, in studying an aerated flow as a continuous medium, some hypotheses about the coupling of the velocities of the components are usually assumed. We will adopt one of those most widely used:

$$\mathbf{v} = \mathbf{v}_{liq} + \mathbf{a}\,. \tag{2}$$

Here a depends on the concentration of the particles, but at a small concentration it can be taken to be constant. Now

$$\mathbf{v}_{mix} = \mathbf{v}_{liq} + \mathbf{a}\frac{\rho}{\rho_{liq} + \rho}\,.$$

If $|\mathbf{v}_{liq}| \gg |\mathbf{a}|$, then the velocity of the mixture differs insignificantly from that of the liquid. Therefore, one can judge the behavior of the velocity of the mixture by the behavior of the velocity of the liquid $\mathbf{v}_{liq}$.

Thus we will study two quantities: ρ and $\mathbf{v}_{liq}\{u, v, w\}$ as functions of space and time. We will consider the problem of the turbulent motion of a mixture over a uniform plane slope.

The coordinate system is selected in the following way: the x axis is along the slope downward, the y axis is perpendicular to the slope upward, the z axis is across the slope. We assume that $\psi = \text{const}$, $\partial/\partial z = 0$, $w = 0$, $\partial/\partial x = 0$. From the equation of the incompressibility of the liquid we have $v = 0$. The general system of turbulent flows yields the following system for ρ and u:

$$\frac{\partial\rho}{\partial t} - \alpha\cos\psi\frac{\partial\rho}{\partial y} = \frac{\partial m}{\partial y}\,,\quad (\rho + \rho_{liq})\frac{\partial u}{\partial t} = \alpha\cos\psi\frac{\partial u}{\partial y} = \rho\, g\sin\psi + \frac{\partial\tau}{\partial y} + m\frac{\partial u}{\partial y}\,. \tag{3}$$

Here the following notation is introduced:

$$m = -\overline{v'\rho'}\,,\quad \tau = -\overline{(\rho + \rho_{\text{liq}})\,u'v'}\,. \tag{4}$$

These quantities are unknown; to close the system, we need hypotheses for them. We can always write

$$\tau = -\nu\frac{\partial(\rho + \rho_{\text{liq}})\,u}{\partial y}\,,\quad m = -k\frac{\partial\rho}{\partial y} \tag{5}$$

and can formulate the hypotheses for ν and k. The majority of the existing hypotheses are constructed in such a manner that $\nu \sim \partial u/\partial y$. This is acceptable if the velocity profile is described by a monotonic function. But if it has an extremum, where $\partial u/\partial y = 0$, which is usually observed in open flows, then according to the hypothesis $\nu \sim \partial u/\partial y$, there is no turbulent mixing at this place. This is unlikely. A number of other assumptions do not give a logarithmic velocity profile near the wall, which would have been confirmed experimentally. Therefore, to study nonstationary motions near the boundary (the velocity profile is nonmonotonic), a new hypothesis is suggested:

$$\nu\,,\ k \approx y\int_{\alpha y}^{\frac{1}{\alpha}y}\left|\frac{\partial u}{\partial y}\right|dy\,. \tag{6}$$

It gives a logarithmic profile of the velocity in a homogeneous medium that moves along a horizontal solid wall at any value of α, and ν and k vanish nowhere. The proportionality factor in formula (6) takes account of the influence of the nonuniformity in the density of the medium in the gravity force field:

$$\nu = C(\alpha)\,y\int_{\alpha y}^{\frac{1}{\alpha}y}\left|\frac{\partial u}{\partial y}\right|dy = f\left(y\,,\ \frac{\partial u}{\partial y}\,,\ \alpha\right). \tag{7}$$

For a homogeneous medium $f = 1$, and for now we will avail ourselves of this approximation for the function f. Equations (3) and (5) with hypothesis (7) form a closed system. We consider the boundary conditions for it.

As is known, the Reynolds equations for turbulent motion can be ignored only when $y > 0$; the point $y = 0$ is singular for them, and boundary conditions cannot be imposed at $y = 0$. If a laminar sublayer is not considered, the boundary conditions on the wall must be specified at some $y = y_0$, where y_0 is the level of roughness (the level where the logarithmic profile of the velocity passes through zero, i.e., $u = 0$). For y_0 there are experimental data for liquid flows over surfaces with different coverings (grass, bushes, asphalt, and so on).

In aerated flows air does not reach the bottom of the flow, and consequently, y_0 will be the same for a flow of a water-air mixture. Therefore we shall avail ourselves of this approach. We consider the following boundary conditions:

for stationary motion ($\partial/\partial t = 0$):

$$\text{when}\quad y = y_0\quad u = 0\,,\ \rho = 0\,;\quad \text{when}\quad y \to \infty\quad \frac{\partial u}{\partial y} = 0\,,\ \rho = 1\,;$$

for nonstationary motion

$$\text{when}\quad y = y_0\quad u = 0\,,\ \rho = 0\,;\quad \text{when}\quad y \to \infty\quad u = 0\,,\ \rho = 1$$

and the following initial data:

$$\text{when}\quad t = t_0\quad u(t_0, y) = U(y)\,,\ \rho(t_0, y) = R(y)\,.$$

Other kinds of boundary conditions are also possible. With account for expressions (4)-(7), system of integrodifferential equations (3) with the given initial and boundary conditions was solved by the difference method.

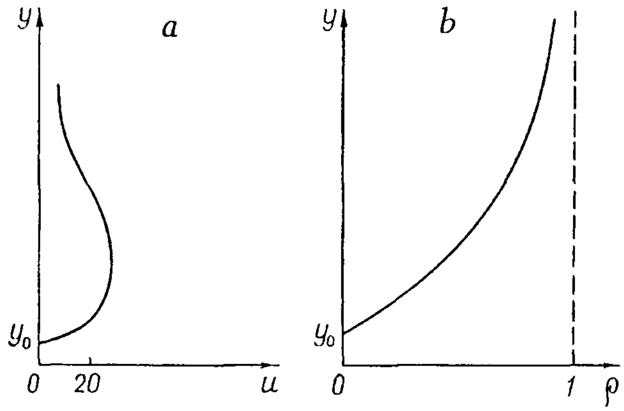

Fig. 2. Result of numerical calculations of the distribution of the velocity (a) and the density (b) over the vertical.

Preliminary calculations were carried out on a computer. In them the following parameters were adopted: $\psi = \pi/6$, $\rho_{liq} = 1$, $\alpha = 0.5$, $y_0 = 0.01$,

$$U(y) = \begin{cases} 20.2\,(y - 0.01)\,, & 0.01 \le y \le 1\,, \\ 20 - 20\,(y - 1)\,, & 1 \le y \le 2\,, \\ 0\,, & y > 2\,, \end{cases} \qquad R(y) = \begin{cases} \frac{1}{0.99}(y - 0.01)\,, & 0.01 \le y \le 1\,, \\ 1\,, & y > 1\,. \end{cases}$$

The qualitative behavior of the solution is given in Fig. 2. From Fig. 2a it is seen that at a certain depth the velocity of the mixture has a maximum, which is located under the conventional free surface. In an aerated flow there is no distinct free surface. From Fig. 2b it follows that conventionally it is located in the zone of y where the density of the mixture differs little from that of air ($\rho = 1$).

Thus, in the present work we proposed a formulation of the problem of determination of the parameters of an aerated flow. A new hypothesis is proposed that allows one to describe Reynolds stresses more realistically. For a homogeneous medium this hypothesis leads to a logarithmic velocity profile, which is well confirmed by experiments.

NOTATION

ρ_{mix}, v_{mix}, density and velocity of the mixture; ρ, ρ_{liq}, density of air and the liquid; v, v_{liq}, velocity of air and the liquid; s, volumetric concentration of air in the mixture, which varies in motion; a, velocity of the uniform slow rise of the particles of air; ψ, angle of inclination of the surface of the slope to the horizonal; m, τ, flux of matter and momentum due to turbulent mixing; u, v, w, components of the vector of liquid velocity; t, time; u', v', w', pulsations of the velocity components; ν, coefficient of momentum; k, coefficient of the flux of matter; α, a certain number smaller than unity; C, function of α; f, function of the coefficient of momentum; U, R, initial functions.

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